

# Moduli Stabilization in Brane Gas Cosmology with Superpotentials

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**ABSTRACT:** In the context of brane gas cosmology in superstring theory, we show why it is impossible to simultaneously stabilize the dilaton and the radion with a general gas of strings (including massless modes) and D-branes. Although this requires invoking a different mechanism to stabilize these moduli fields, we find that the brane gas can still play a crucial role in the early universe in assisting moduli stabilization. We show that a modest energy density of specific types of brane gas can solve the overshoot problem that typically afflicts potentials arising from gaugino condensation.

**KEYWORDS:** Brane Gas Cosmology.

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## 1. Introduction

A major success of brane gas cosmology (BGC) is the utilization of stringy effects to explain the origin of the hierarchy of dimensions. In the seminal proposal of Brandenberger and Vafa [1], it was argued that in the early universe all directions could fluctuate about the self-dual radius due to the presence of both winding and momentum modes. The argument asserts that strings will generically intersect in  $(3+1)$ -dimensional subspaces, so that such a subspace will lose its winding and subsequently expand into the large directions we observe today. This scenario was mathematically realized by Tseytlin and Vafa in the context of dilaton gravity [2], and has since been extended to include the effects of a gas of  $Dp$ -branes, where Alexander, Easson, and Brandenberger [3] argued that such a gas would result in a

hierarchy of extra dimensions. Namely the original 9-dimensional spatial manifold should decompactify into a hierarchical product space of  $\mathcal{T}_4 \times \mathcal{T}_2 \times \mathcal{T}_3$ .

Subsequent investigations indicate that wound strings provide a mechanism for isotropization [4] and stabilization [5] of the compact dimensions, and that the mechanism works on toroidal orbifolds [6]. The framework for these results is usually the low energy effective action of type IIA string theory, where the salient differences from general relativity are a massless dilaton and the dynamics of extra dimensions. These differences lead to the result that negative pressure in the compact directions, due to wound strings, results in contraction, not acceleration. The dilaton is assumed to have no potential other than that which is induced by its coupling to the bulk string frame Lagrangian  $e^{-2\phi}(R + (\nabla\phi)^2)$  and possible D-brane sources; most successes of BGC rely on the dynamical running of the dilaton toward weak string coupling,  $g_s \ll 1$ .

On the other hand one would like to stabilize the dilaton at a value where  $g_s$  is still large enough to be consistent with gauge coupling unification [7]. Moreover if  $g_s$  becomes too small, the interactions between strings become too weak to allow the annihilation of winding modes in three dimensions, where the space should be allowed to grow [8]. (See also [9] for a discussion similar issues.) Rather there is only a window of finely-tuned initial conditions consistent with three dimensions ultimately growing to be large. A third reason that the dilaton must not continue to roll to arbitrarily small values is the constraint from fifth force experiments and null searches for time variation of physical constants [10]; these preclude the dilaton from continuing to evolve at late times.

For these reasons it is imperative to reconcile brane gas cosmology with the stabilization of both the radion and the dilaton. In [11] it was shown that using just the string winding and momentum modes this is not possible. We therefore first investigate whether by including more general string and brane states one can achieve such a stabilization. (A similar but less general analysis has been done in [12].)

Our result, in the context of superstring theories, is that a general gas of D-branes and strings cannot stabilize both moduli, although they can stabilize one linear combination of them. However, the brane gas can still play an important role in the process of stabilizing both fields, due to the overshoot problem [13]. A much-studied mechanism for stabilizing the radion involves adding racetrack potentials coming from gaugino condensation (and possibly an antibrane [14]). The Minkowski minimum of these potentials is typically separated from a runaway (decompactification) direction by a very small barrier, which would always be overcome by the inertia of the fields if their initial conditions were not finely tuned to be close to the desired minimum. One of our main observations is that a gas of brane winding modes can very robustly solve this problem by slowing down the modulus as it rolls down its steep potential.

Our plan is as follows: in Section 2.1 we describe the BGC scenario and motivate the dimensional reduction procedure to obtain a  $d$ -dimensional theory of gravity with two scalar fields (the dilaton and radion), with an effective potential coming from the brane gas. In Section 2.3 we discuss some of the features of the effective potential; namely, we show that provided the dilaton is stabilized by some other mechanism, branes can stabilize all the extra dimensions. Section 3 presents our no-go theorem showing that under the

given assumptions, there exists an unstabilized direction in the moduli space of the dilaton and radion no matter what modes are included in the gas of D-branes and strings. In particular we also show that the presence of massless F-string modes do not help in lifting the runaway direction. In section 4 we consider the combined effect of the brane gas with a superpotential, such as would arise from gaugino condensation and antibranes, and show that the brane gas can provide a remedy for the overshoot problem. We give our conclusions in section 5. Technical details are given in the appendices.

## 2. Effective Brane Gas Cosmology

### 2.1 Supergravity coupled to Strings and Brane Sources

A starting point for BGC is type IIA string theory compactified on a 9-dimensional toroidal background, which may be thought of as the result of compactifying  $M$ -theory on  $S^1$ . The low-energy bulk effective action of this theory is given by

$$S_{IIa} = \frac{1}{2\kappa^2} \int d^D x \sqrt{-G} e^{-2\phi} \left( R + 4G^{MN} \nabla_M \phi \nabla_N \phi - \frac{1}{12} H_{\mu\nu\alpha} H^{\mu\nu\alpha} \right), \quad (2.1)$$

where  $G$  is the determinant of the ten-dimensional background metric  $G_{\mu\nu}$ ,  $\phi$  is the dilaton,  $H$  is the field strength corresponding to the bulk antisymmetric tensor field  $B_{\mu\nu}$ , and  $\kappa$  is the D-dimensional Newton's constant. For simplicity we ignore any flux contributions, and take  $H = 0$ . We envision this analysis to apply in the late-time era of BGC [1, 3, 6, 15], an epoch where the extra, compact dimensions are expected to be isotropized [4], and winding modes in the large directions have annihilated. Thus, we consider a spacetime consisting of a flat,  $d$ -dimensional FRW universe, and an isotropic compact subspace of  $n$  extra dimensions

$$ds^2 = G_{MN} dX^M dX^N = g_{\mu\nu} dx^\mu dx^\nu + b^2(t) \gamma_{mn} dy^m dy^n \quad (2.2)$$

$$= -dt^2 + a^2(t) dx_i dx^i + b^2(t) dy_m dy^m, \quad i \in \{1, \dots, d\}, m \in \{1, \dots, n\} \quad (2.3)$$

where  $y^m$  are the coordinates of the  $n$  extra dimensions. The total action comprises the above bulk action (2.1) and the action of all matter present. Sources are included by adding matter terms for both the strings ( $\rho_s$ ) and Dp-branes ( $\rho_p$ ). Owing to the different world-sheet couplings between the dilaton and the branes and strings, the matter action has the form

$$S_m = - \int d^D x \sqrt{-G} \left( \rho_s + e^{-\phi} \rho_p \right) \quad (2.4)$$

$$T_{MN} = - \frac{2}{\sqrt{-G}} \frac{\delta S_m}{\delta G^{MN}}. \quad (2.5)$$

We continue the construction of late-time BGC by considering separate species of strings and branes, each possibly having excited momentum (in the case of branes also known as “vibrational modes” [16]) in the large or compact subspaces, but having winding

modes only along the compact directions. Then one can show (see appendix B) that the stress energy tensor for the strings and branes simplifies to

$$-T_0^0 = \rho_s + e^{-\phi} \rho_p = \sum_i \left[ \rho_i e^{-\alpha_i \phi} a^{-d(1+\omega_i)} b^{-n(1+\hat{\omega}_i)} \right] \quad (2.6)$$

$$T_b^a = P \delta_b^a = \sum_i \omega_i \left[ \rho_i e^{-\alpha_i \phi} a^{-d(1+\omega_i)} b^{-n(1+\hat{\omega}_i)} \right] \delta_b^a \quad (2.7)$$

$$T_n^m = p \delta_n^m = \sum_i \hat{\omega}_i \left[ \rho_i e^{-\alpha_i \phi} a^{-d(1+\omega_i)} b^{-n(1+\hat{\omega}_i)} \right] \delta_n^m. \quad (2.8)$$

In the preceding expressions the summation is performed over the relevant modes contributing to the gas of strings and branes,  $P$  and  $p$  being the sum-total pressure along the large and compact directions respectively.  $\alpha_i = 0$  for string sources,  $\alpha_i = 1$  for brane sources, and  $\rho_i$  is the initial energy density for a particular mode, with effective equation of state  $p_i = \hat{\omega}_i \rho_i$ ,  $P_i = \omega_i \rho_i$ . The values of  $\omega$  and  $\hat{\omega}$  depend on the specific type of mode, dimensionality of the branes and the number of large and extra dimensions (see table 1), but the important thing is that all the known modes can be described by these quantities.

Variation of the action (2.1) together with the matter action (2.4) and metric ansatz (2.3) results in the system of equations

$$-d \left( \frac{\dot{a}(t)}{a(t)} \right)^2 - n \left( \frac{\dot{b}(t)}{b(t)} \right)^2 + \dot{\varphi}^2 = e^\varphi E \quad (2.9)$$

$$\frac{d}{dt} \left( \frac{\dot{a}(t)}{a(t)} \right) - \dot{\varphi} \frac{\dot{a}(t)}{a(t)} = \frac{1}{2} e^\varphi P \quad (2.10)$$

$$\frac{d}{dt} \left( \frac{\dot{b}(t)}{b(t)} \right) - \dot{\varphi} \frac{\dot{b}(t)}{b(t)} = \frac{1}{2} e^\varphi p \quad (2.11)$$

$$\ddot{\varphi} - d \left( \frac{\dot{a}(t)}{a(t)} \right)^2 - n \left( \frac{\dot{b}(t)}{b(t)} \right)^2 = \frac{1}{2} e^\varphi E, \quad (2.12)$$

where we have introduced the shifted dilaton as  $\varphi \equiv 2\phi - d \frac{\dot{a}(t)}{a(t)} - n \frac{\dot{b}(t)}{b(t)}$  (recall that  $d = 3$  and  $n = 6$ ), and a dot denotes differentiation with respect to time. Eqs. (2.9-2.12) are the string frame, or dilaton-gravity, equations of motion. Equation (2.9) is the 0-0 Einstein equation; notice that in the string frame the kinetic term for the (shifted) dilaton contributes to the energy with apparently the wrong sign—this is due to the nonminimal coupling between the Ricci Scalar and the dilaton. The spatial components of the Einstein equations (2.10-2.11) show that the acceleration of the scale factor is proportional to the pressure, and thus the negative-pressure winding modes lead to contraction—this is the key ingredient of the Brandenberger-Vafa mechanism. Eq. (2.12) is the dilaton equation of motion. Equation (2.9) is not dynamical, but is rather an equation of constraint, which can be used to determine the initial dilaton velocity to be

$$\dot{\varphi} = \pm \sqrt{e^\varphi E + d \left( \frac{\dot{a}(t)}{a(t)} \right)^2 + n \left( \frac{\dot{b}(t)}{b(t)} \right)^2}. \quad (2.13)$$

It is customary to choose the negative solution since the string coupling  $g_s = e^\phi$  then evolves toward weak coupling, where a perturbative description is valid. Since all the terms under the square root are positive, the dilaton cannot bounce.

The rolling of the dilaton, although important for the BV mechanism and the stabilization of the moduli fields, may also be deleterious to the BGC scenario. In [8], Easter, Greene, Jackson, and Kabat show that if the dilaton rolls too quickly, winding-mode annihilation may be suppressed, so that dynamical evolution leading to three large spatial dimensions is not favoured. The rolling of the dilaton also implies evolution of volume of the compact space. A conformal transformation on the metric may absorb the  $\phi$ - $R$  coupling term, but this means the Einstein frame scale factors get additional time dependence from  $\phi(t)$ . This problem has typically been set aside (on the assumption that the dilaton will be stabilized at a later time) in discussions of stabilization of the extra dimensions [5]. However, in order to have a complete and consistent picture in the framework of brane gas cosmology one indeed needs to address the issue of stabilizing the dilaton along with radion stabilization, and this is what we devote the next few subsections to.

The preceding observations also stress the utility of viewing gravity from the point of view of the four-dimensional Einstein frame, which is more intuitive than the 10D string frame. An effective four-dimensional action is achieved by conformally absorbing the dilaton, integrating out the extra dimensions, and performing a second conformal transformation to absorb the scale factor of the extra dimensions. The result is a minimally-coupled theory of BGC, where the original string and brane sources act as an effective potential for both the radion and dilaton fields. This approach was first advocated in [17] to study stabilization of extra dimensions in the presence of hydrodynamical fluids and was used to study string winding and momentum modes in [11]. We now generalize the analysis to include all possible string and brane sources.

## 2.2 Effective Potential

Upon performing dimensional reduction on both the string and brane sources, we obtain general relativity coupled to two scalar fields, the dilaton and radion, with an effective potential coming from the brane gas. As outlined in Appendix A, a string/brane source whose energy density behaves as  $\rho = \rho_i e^{-\alpha_i \phi} \bar{a}^{-d(1+\omega_i)} b^{-n(1+\hat{\omega}_i)}$ , with equations of state  $\omega_i$  and  $\hat{\omega}_i$  in the  $d$  large and  $n$  compact directions respectively, provides an effective potential in  $d+1$  dimensions

$$\begin{aligned}
V_{\text{eff},i} &= \rho_i e^{2\nu_i \psi} e^{2\mu_i \varphi} \bar{a}^{-d(1+\omega_i)} \\
\nu_i &= \frac{1}{2} \left( -\hat{\omega}_i + \frac{d}{d-1} \left( \omega_i - \frac{1}{d} \right) \right) \sqrt{\frac{(d-1)n}{(d+n-1)}} \\
\mu_i &= \frac{1}{2} \left( -d\omega_i - n\hat{\omega}_i + 1 - \alpha_i \frac{d+n-1}{2} \right) \sqrt{\frac{1}{d+n-1}}
\end{aligned} \tag{2.14}$$

This is expressed in terms of the canonically normalized moduli  $\psi$  and dilaton  $\varphi$  fields, and the Einstein-frame scale factor  $\bar{a}$  of the  $d$  large directions.  $\alpha_i$  parametrizes string ( $\alpha_i = 0$ ) or brane ( $\alpha_i = 1$ ) contributions, and  $\rho_i$  is the initial energy density of the  $i$ th component

of the brane gas. We work in Planck units,  $M_{pl}^{-2} = 8\pi G_N = 1$ . The net effective potential will comprise several contributions of the form (2.14), depending on the type of excited modes; Appendix B discusses the equations of state, and the coefficients  $\mu_i$ ,  $\nu_i$  for the various string and brane sources and the results are summarized in Table 1.

source	$E \propto a^{-d\omega} b^{-n\hat{\omega}}$	$\omega$	$\hat{\omega}$	$\mu_i$	$\nu_i$
$d = 3$					
general string	$a^{-d\omega} b^{-n\hat{\omega}}$	$\omega$	$\hat{\omega}$	$\frac{-3\omega - n\hat{\omega} + 1}{\sqrt{4(n+2)}}$	$-\frac{(\hat{\omega} + \frac{1}{2}(1-3\omega))\sqrt{n}}{\sqrt{2(n+2)}}$
general brane	$a^{-d\omega} b^{-n\hat{\omega}}$	$\omega$	$\hat{\omega}$	$-\frac{3\omega + n\hat{\omega} + \frac{n}{2}}{\sqrt{4(n+2)}}$	$-\frac{(\hat{\omega} + \frac{1}{2}(1-3\omega))\sqrt{n}}{\sqrt{2(n+2)}}$
wound string	$a^0 b^1$	0	$\frac{-1}{n}$	$\frac{1}{\sqrt{n+2}}$	$\frac{1 - \frac{n}{2}}{\sqrt{2n(n+2)}}$
wound brane	$a^0 b^p$	0	$\frac{-p}{n}$	$\frac{(p - \frac{n}{2})}{2\sqrt{n+2}}$	$\frac{p - \frac{n}{2}}{\sqrt{2n(n+2)}}$
string momentum	$a^0 b^{-1}$	0	$\frac{1}{n}$	0	$-\sqrt{\frac{n+2}{8n}}$
brane momentum	$a^0 b^{-1}$	0	$\frac{1}{n}$	$-\frac{1}{4}\sqrt{n+2}$	$-\sqrt{\frac{n+2}{8n}}$
$d = 3, n = 6$					
wound string	$a^0 b^1$	0	$\frac{-1}{6}$	$\frac{1}{\sqrt{8}}$	$-\frac{1}{2\sqrt{6}}$
wound brane	$a^0 b^p$	0	$\frac{-p}{n}$	$\frac{p-3}{2\sqrt{8}}$	$\frac{p-3}{4\sqrt{6}}$
string momentum	$a^0 b^{-1}$	0	$\frac{1}{n}$	0	$\frac{-1}{\sqrt{6}}$
brane momentum	$a^0 b^{-1}$	0	$\frac{1}{n}$	$-\frac{1}{\sqrt{2}}$	$\frac{-1}{\sqrt{6}}$

**Table 1:** A summary of the radion and dilaton couplings in the effective potential due to various species of gas. The spatial background consists of  $d$  large and  $n$  compact directions, with equations of states  $\omega$  and  $\hat{\omega}$  respectively.

### 2.3 Radion Stabilization

To understand the effects of string and brane sources in late-time BGC, we now specialize to the case of three large directions ( $d = 3$ ) with winding modes only in the compact dimensions. First suppose that brane sources are not present, so the effective potential for the system is given by contributions from strings alone—this emulates the setup of [1, 5, 11, 15, 18]. Three representative species of strings are considered, namely,  $W$ : strings with winding numbers in the compact direction ( $\omega = 0$ ,  $\hat{\omega} = -\frac{1}{n}$ ),  $M_6$ : momentum excitations in the compact directions ( $\omega = 0$ ,  $\hat{\omega} = \frac{1}{n}$ ), and  $M_3$ : momentum in the large directions ( $\omega = \frac{1}{d}$ ,  $\hat{\omega} = 0$ ). Summing contributions (2.14), we obtain

$$V_s(\bar{a}, \varphi, \psi) = \rho_W e^{(1-\frac{n}{2})\sqrt{B}\psi} e^{\sqrt{\frac{A}{2}}\varphi} \bar{a}^{-3} + \rho_{M_6} e^{-(1+\frac{n}{2})\sqrt{B}\psi} \bar{a}^{-3} + \rho_{M_3} \bar{a}^{-4}, \quad (2.15)$$

where  $\rho_W$ ,  $\rho_{M_3}$ ,  $\rho_{M_6}$  parametrize the initial energy densities of the three kinds of components,  $B = \frac{2}{n(n+2)}$ , and  $A = \frac{2}{n+2}$ . Let us assume that the dilaton has been stabilized by an external potential and consider the effect of the string gases on the unstabilized radion. Taking the dilaton VEV to be  $\phi = 0$  and ignoring the  $M_3$  momentum modes, which anyway

gets redshifted by the expansion of the universe, the string gas effective potential (2.15) becomes

$$V_s(\bar{a}, \psi) = \bar{a}^{-3} \left[ \rho_N e^{(1-\frac{n}{2})\sqrt{B}\psi} + \rho_M e^{-(1+\frac{n}{2})\sqrt{B}\psi} \right]. \quad (2.16)$$

Battefeld and Watson point out [11] that this is a stable potential for  $\psi$  only if the number of extra dimensions is  $n = 1$ , in which case it reduces to  $V(\bar{a}, \psi = 0) \sim \frac{1}{\bar{a}^3}$ . This can be considered a source of dark matter, similar to the string-inspired example of Gubser and Peebles [19]. However, in the case of  $n > 2$ , [11] points out that the effective potential behaves as  $V(\bar{a}, \psi) \sim e^{-\alpha\psi}/\bar{a}^3$ , so that the radion also runs away to  $\infty$ . Since  $n = 6$ , one sees that the presence of strings cannot stabilize the dilaton or the radion.

We note that in [20, 21] massless string states were invoked to obtain stabilization of the moduli. However, the former are not present in the type II string (being removed by the GSO projection). Although, they are present at the self-dual radius in the heterotic string,<sup>1</sup> additionally, [21] requires quantized modes of the D-string to achieve complete stabilization of all moduli. It is not clear to us that the D-string can be quantized in the same way as the fundamental string.

Let us therefore consider whether extending the analysis of [11] to the case of general brane sources can solve the problem of moduli stabilization. Consider the contributions to  $V_{\text{eff}}$  coming from  $p$ -branes wrapping the compact dimensions ( $\omega = 0$ ,  $\hat{\omega} = -\frac{p}{n}$ , denoted  $\tilde{N}$ ), and momentum modes in the compact dimensions ( $\omega = 0$ ,  $\hat{\omega} = \frac{1}{n}$ , denoted  $\tilde{M}$ ). The net effective potential from equation (2.14) is

$$V_p(\bar{a}, \varphi, \psi) = \bar{a}^{-3} \left[ \rho_{\tilde{N}} e^{(p-\frac{n}{2})\sqrt{B}\psi} e^{(p-\frac{n}{2})\sqrt{\frac{A}{2}}\varphi} + \rho_{\tilde{M}} e^{-(1+\frac{n}{2})\sqrt{B}\psi} e^{-(1+\frac{n}{2})\sqrt{\frac{A}{2}}\varphi} \right] \quad (2.17)$$

This scenario is similar to those analyzed in [3, 4, 12, 16, 18]. As we will now explore, the result of including a gas of branes is the improved stability of the radion. Inspection of (2.17) reveals that provided  $p > \frac{n}{2}$ , all internal directions will be stabilized, since there are both rising and falling exponentials depending on  $\psi$ :

$$V_p(\bar{a}, \psi) = \rho_{\tilde{N}} e^{(p-\frac{n}{2})\sqrt{B}\psi} \bar{a}^{-3} + \rho_{\tilde{M}} e^{-(1+\frac{n}{2})\sqrt{B}\psi} \bar{a}^{-3}. \quad (2.18)$$

where again we have assumed  $\varphi = 0$ . This has a nontrivial minimum close to  $\psi = 0$  provided that  $p > \frac{n}{2}$ . Since string theory requires  $n = 6$ , the presence of  $(p > 3)$ -branes in the compact directions will stabilize the moduli.

However a more detailed analysis may be necessary to realize these stability conditions: According to the heuristic argument of Alexander, Brandenberger, and Easson [3], winding modes will generically intersect in  $2p+1$  dimensions, so that only objects with  $p \leq 2$  should remain wound in the 6 compact directions. In this case, the stability requirement will not be satisfied. On the other hand, a quantitative investigation should account for the larger phase space once the  $\bar{a}$  directions have grown large, thus decreasing the probability of annihilation, and perhaps leaving some extended objects with  $p > \frac{n}{2}$ . As well, such an

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<sup>1</sup>We thank Subodh Patil for discussions on this point.

analysis should be carried out within the product space  $\mathcal{T}_4 \times \mathcal{T}_2 \times \mathcal{T}_3$  argued for by [3], not the  $\mathcal{T}_n \times \mathcal{T}_3$  topology we have considered. We do note though that, as long as the shape moduli is frozen, the exact shape of the tori does not matter and our stability analysis for the volume still applies.

### 3. No-Go Result in Type II String Theory

In the previous section we saw that the radion can be stabilized by a brane gas when the dilaton is assumed to be fixed; similarly one can show the corresponding result when the roles of the dilaton and radion are interchanged. One may naturally wonder whether both of these moduli can be simultaneously stabilized using the most general combination of string and brane sources. As we now show, this is impossible to do with the conventional (winding, momentum or oscillator) string and brane excitations. The argument is made in two steps, starting first with gases where each string or brane has only one kind of excitation (“simple states”), although different species of strings or branes are allowed to co-exist. We then extend the argument to the more general case where individual components of the gas have more than one kind of excitation (“mixed states”).

#### 3.1 “Simple States”

We consider the situation when the strings/branes have nontrivial wrapping of only some extra dimensions, i.e. it doesn’t wrap the large dimensions. They thus appear point-like to 4D observers and redshift like nonrelativistic dust,  $\bar{a}^{-3}$ , corresponding to  $d = 3$ ,  $\omega = 0$  in (2.14). The equations of motion for the radion and the dilaton in the presence of such sources are

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{\partial V_{\text{eff}}(\varphi, \psi)}{\partial \varphi} \quad (3.1)$$

$$\ddot{\psi} + 3H\dot{\psi} = -\frac{\partial V_{\text{eff}}(\varphi, \psi)}{\partial \psi} \quad (3.2)$$

with

$$V_{\text{eff}}(\varphi, \psi) = \bar{a}^{-3} \sum_i \rho_i e^{2\mu_i \varphi + 2\nu_i \psi} \quad (3.3)$$

where the sum runs over all possible string and brane states. As summarized in Appendix B, the exponents  $\mu_i, \nu_i$  depend only on the effective equation of state parameter  $\hat{\omega}$  along the extra dimensions (2.14), and the coupling exponent  $\alpha$  of these states to the dilaton in the string frame:

$$S_{\text{gas},i} = - \int d^D x \sqrt{-g} e^{-\alpha_i \phi} \rho_i b^{-n(1+\hat{\omega}_i)} a^{-3} . \quad (3.4)$$

where  $\alpha = 0$ , for the fundamental strings since the Nambu-Goto action does not contain any dilaton coupling, while for branes  $\alpha = 1$ , originating from the dilaton coupling in the DBI action.

After performing the conformal transformations involving the radion and the dilaton (see Appendix A) the above action gives rise to the effective potential (2.14) in the 4D

Einstein frame with

$$\mu_i = \frac{1}{2\sqrt{n+2}} \left[ 1 - \alpha_i - n(\hat{\omega}_i + \frac{\alpha_i}{2}) \right]; \quad \nu_i = -\sqrt{\frac{n}{2(n+2)}} \left( \hat{\omega}_i + \frac{1}{2} \right) \quad (3.5)$$

As noted earlier, the value of  $\hat{\omega}$  depends upon whether the mode in question has winding, momentum or string oscillations. To analyze the stability of a potential which is a sum over such modes, we will use the technique of [22]: we identify the directions in the  $\varphi$ - $\psi$  plane in which there exists a rising exponential contribution. If such directions are sufficiently numerous, the system is completely stabilized. An exponential of the form  $\rho_i e^{2\mu_i \varphi + 2\nu_i \psi}$  rises most steeply along the direction  $\cos \theta_i \hat{\varphi} + \sin \theta_i \hat{\psi}$  where  $\tan \theta_i = \nu_i / \mu_i$ . In the range

$$\theta = (\theta_i - \pi/2, \theta_i + \pi/2) \quad (3.6)$$

there is a rising potential (wall) while along the other half-plane the potential asymptotically falls to zero. Since our potential is a sum of exponentials, it is clear that:

- (I) There can be at most a single local minimum and no local maxima.
- (II) Such a minimum exists only if the potential grows in any direction away from the minimum. Thus there must exist angles  $\theta_i$  for which the ranges of angles in (3.6) cover the entire plane.

By looking at the different directions of steepest ascent of the exponentials it is easy to verify whether (II) is satisfied. Curiously, for brane sources ( $\alpha_i = 1$ ) we find a result that is specific to  $d = 3$  large dimensions: the direction of steepest ascent is the same (modulo  $\pi$ ) for winding, momentum or any other modes. For the winding modes, it is given by

$$\tan \theta = \frac{\nu_i}{\mu_i} = \sqrt{\frac{2}{n}} \Rightarrow \theta = \frac{\pi}{6} + \begin{cases} 0, & p > 3 \\ \pi, & p < 3 \end{cases} \quad (3.7)$$

while for momentum modes

$$\theta = \frac{\pi}{6} \quad (3.8)$$

for all  $p$ , as illustrated in figure 1. Thus using, say, a gas of 2- and 6-branes in type IIA theory, or a gas of 1 and 5-branes in IIB theory, one could stabilize all directions of the  $\varphi$ - $\psi$  plane except for those orthogonal to the direction of steepest ascents, that is,  $\theta = \frac{\pi}{6} \pm \frac{\pi}{2}$  (see figure). Along these directions the potential is flat, and there is a zero mode.

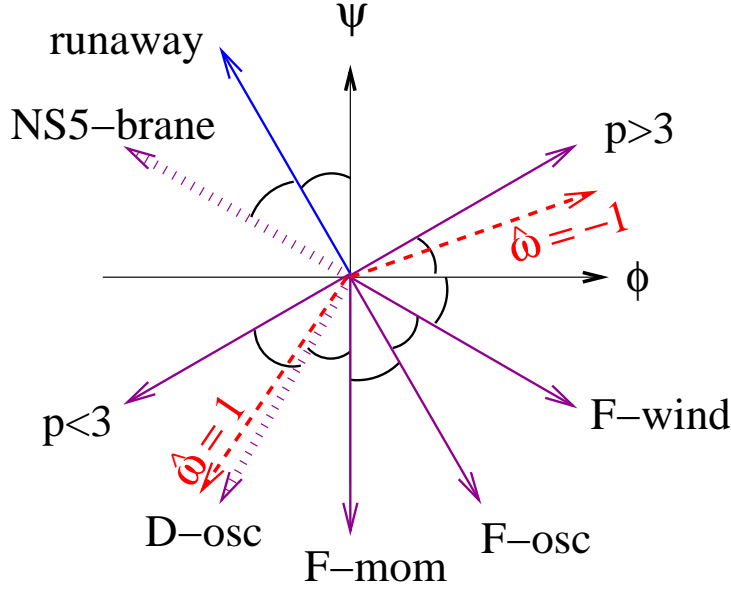
To lift the flat direction, one has to incorporate fundamental string sources, with  $\alpha_i = 0$ . According to Table 1 for F-strings, the angle

$$\tan \theta = \frac{\nu_i}{\mu_i} = \sqrt{\frac{2}{n}} \frac{\hat{\omega}_i + \frac{1}{2}}{\hat{\omega}_i + \frac{1}{n}} \quad (3.9)$$

depends on  $\hat{\omega}_i$ , and so the analysis has to be done separately for each case. For the string momentum, oscillatory and winding modes,  $\hat{\omega} = 1/n$ ,  $0$  and  $-1/n$  respectively. From (3.5) one then finds

$$\theta_{\text{mom}} = -\frac{\pi}{2}, \quad \theta_{\text{osc}} = -\frac{\pi}{3}, \quad \theta_{\text{wind}} = -\frac{\pi}{6} \quad (3.10)$$

All of these modes provide an ascending potential in the direction of  $\theta = -\frac{\pi}{3}$ , which is the flat direction when only brane sources are present, but not along the opposite direction



**Figure 1:** Directions of steepest ascent in the  $\phi$ - $\psi$  plane for contributions from different brane gas sources, described in the text. Long dashed lines are for the hypothetical F-string sources with  $\hat{\omega} = \pm 1$ . All marked angles are  $30^\circ$ . Short dashed lines are for the sources described in section 3.3.

(see figure 1). Thus after including the string modes, there is no longer a zero mode, but there is a runaway direction along  $\theta_{\text{run}} = \frac{2\pi}{3}$ .

In fact we can make an even stronger statement. Suppose that there are other string sources we may be unaware of; nevertheless their equations of state should satisfy the weak energy condition  $-1 \leq \hat{\omega} \leq 1$ . Using the Table 1 entries for general string sources and varying  $\hat{\omega}$  over this range gives an angle of steepest ascent in the range

$$\pi - \tan^{-1} \frac{3\sqrt{6}}{5} \leq \theta \leq \tan^{-1} \frac{\sqrt{6}}{7} \quad \rightarrow \quad -124^\circ \lesssim \theta \lesssim 19^\circ \quad (3.11)$$

which again fails to lift the  $\theta_{\text{run}}$  direction, as shown in figure 1.

An alternative understanding of the moduli instability can be directly inferred by a reparametrization of the effective potential (2.14) in terms of new fields  $\chi = \sqrt{B}\psi + \sqrt{\frac{A}{2}}\varphi$  and  $\eta = \sqrt{\frac{A}{2}}\psi - \sqrt{B}\varphi$ . The result for an arbitrary string and brane gas with three large directions and  $\omega = 0$  is

$$V_{\text{eff},i}(\alpha = 0) = \rho_i e^{(-n(\hat{\omega} + \frac{1}{4}) + \frac{1}{2})\chi} e^{-(\frac{n}{2} + 1)\frac{\eta}{2}} \bar{a}^{-3} \quad (\text{string}) \quad (3.12)$$

$$V_{\text{eff},i}(\alpha = 1) = \rho_i e^{-n(\hat{\omega}_i + \frac{1}{2})\chi} \bar{a}^{-3} \quad (\text{brane}) \quad (3.13)$$

Through a combination of sources it is possible to stabilize the  $\chi$  mode; however string sources will only cause  $\eta$  to grow, and a brane gas does not couple to  $\eta$ . Thus  $\hat{\eta} = \sqrt{\frac{A}{2}}\hat{\psi} - \sqrt{B}\hat{\varphi}$  is the unstable direction in field space, in terms of the unit vectors  $\hat{\varphi}$ ,  $\hat{\psi}$ .

This direction corresponds to the line

$$\psi = -\sqrt{\frac{A}{2B}} \varphi = -\sqrt{\frac{n}{2}} \varphi , \quad (3.14)$$

which coincides precisely with the principal runaway direction identified previously.

The above result is consistent with ref. [18], which used a perturbation analysis in the BGC scenario to show that the inclusion of branes alone is not enough to stabilize both the dilaton and moduli fields. Thus some other potential is needed to stabilize one of the moduli. Given such a potential, BGC does provide a mechanism of stabilizing the other degree of freedom provided that branes with  $p > \frac{n}{2}$  are present.

The factorization of the effective potential is a coincidence of having  $d = 3$  large dimensions, as can be seen from the nontrivial dependence on  $d$  in eq. (2.14). For scenarios other than  $d = 3$ , the gas of strings and branes is able to stabilize both fields.

Finally, we note that in the above analysis we did not consider branes or strings which wrap some of the large three dimensions ( $\omega < 0$ ); these do not give any additional leverage for stabilizing the radion.

### 3.2 “Mixed States” and Massless Modes

So far we have only considered states which are purely oscillatory, winding or momentum modes. More generally, strings could have a combination of such excitations. Can such mixed modes help in stabilizing the moduli? The answer, unfortunately is no. For massive modes the reasons are similar to the case of the simple states. The massless modes<sup>2</sup> have to be analyzed separately but they do not alter the conclusions.

In the string frame, the string spectrum is

$$m_F^2 = \frac{m^2}{b^2} + N_{\text{osc}} M_s^2 + w^2 b^2 M_s^4 \quad (3.15)$$

where  $M_s$  is the string scale and the three terms on the right hand side correspond to the momentum, oscillatory and winding pieces respectively. The source action for the strings (3.4) then becomes

$$S_{\text{str}} = \int d^D x \sqrt{-g} a^{-3} b^{-n} \sqrt{m_F^2(b) + p^2} \quad (3.16)$$

where  $p$  is the momentum along the non-compact directions. After performing the dimensional reduction and conformal redefinitions, as usual we find that the effective potential for the canonical radion and dilaton coming from a gas of such states is given by

$$V_{\text{eff}}(\varphi, \psi) = \rho = n E(\varphi, \psi) \quad (3.17)$$

where  $n$  is the number density, and  $E(\psi, \phi)$  is the energy of these states which depends on both the dilaton and the radion. Since we already know how the exponents look like for

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<sup>2</sup>Although, in the type II theory these are excitations of the tachyon which are removed by the GSO projection, such massless winding states are allowed in the heterotic string theory.

individual momentum, winding and oscillatory modes, it is easy to see that for the more general case the energy is just given by

$$E(\psi, \varphi) = \sqrt{M_s^2 [m^2 e^{4(\mu_m \varphi + \nu_m \psi)} + w^2 e^{4(\mu_w \varphi + \nu_w \psi)} + N_{osc} e^{4(\mu_o \varphi + \nu_o \psi)}] + p^2} \equiv \sqrt{m_F^2 + p^2} \quad (3.18)$$

with

$$\mu_m = 0 ; \mu_o = \frac{1}{2\sqrt{8}} \text{ and } \mu_w = \frac{1}{\sqrt{8}}$$

and

$$\nu_m = -\frac{1}{\sqrt{6}} ; \nu_o = \frac{\sqrt{3}}{2\sqrt{8}} \text{ and } \nu_w = -\frac{1}{2\sqrt{6}} \quad (3.19)$$

First let us focus on the massive modes, for which one can ignore the momentum  $p$ . The key observation is that since  $N_{osc} \geq 0$  for massive modes, the effective potential obtained in the Einstein frame must still satisfy conditions (I) and (II) above in order to have a local minimum. Since again such potentials can have at most one minimum and no maximum, if there exists a minimum, the potential has to keep rising along any direction as one tends towards infinity. Thus to determine whether there is a minimum, it suffices to investigate the behaviour at infinity in the  $\varphi$ - $\psi$  plane. Going far enough toward  $\infty$  along a generic direction, one of the three terms in (3.15) will dominate, and then our previous analysis applies, which assumed the presence of only one term in a given source. In the special direction where  $b$  remains constant, no one term dominates, but they all remain proportional to each other, behaving like a single term, so again the previous analysis remains valid.

Next let us focus on the massless modes, for example, the ones considered in [21]. In this case  $N_{osc} < 0$ ; and depending on the winding and momentum quantum numbers one could have  $m_F^2 \sim (m/b - wbM_s^2)^2$  leading to a minimum at  $b^2 = (m/w)M_s^{-2}$  (which is the self-dual radius when  $m/w = 1$ ) [21]. For these modes one can easily verify that the mass function can be cast as

$$m_F^2(\varphi, \psi) \sim e^{\sqrt{2}\varphi'} \left( e^{\frac{\psi'}{\sqrt{6}}} - e^{-\frac{\psi'}{\sqrt{6}}} \right)^2 \quad (3.20)$$

where

$$\psi' = \frac{\sqrt{3}}{2}\varphi + \frac{1}{2}\psi \quad (3.21)$$

(as one can find by carefully tracing back the conformal transformations) is really the string frame radion and

$$\varphi' = \frac{1}{2}\varphi - \frac{\sqrt{3}}{2}\psi \quad (3.22)$$

is the orthogonal direction. As one can see, the mass (3.20) and the potential have a minimum at  $\psi' = 0$  and hence the massless states stabilize the  $\psi'$  direction, as argued in [21]. However,  $\psi'$  also precisely coincides with the direction that could be fixed just with winding branes. Thus we are still left with the orthogonal runaway direction ( $\varphi' \rightarrow -\infty$ ) that we found earlier.

### 3.3 Exotic States

We have seen so far that ordinary D-brane and string states are unable to stabilize both the radion and the dilaton simultaneously. We now briefly discuss how stabilization might be achieved using some less conventional kinds of branes.

One kind of exotic state which has been considered [21] are quantized D-string modes. Whether it is justified to derive these from the Nambu-Goto action like for F-strings seems doubtful, since the D-string is a solitonic object, but for completeness we have derived the exponents corresponding to the different D-string modes<sup>3</sup>. Although oscillator excitations do provide a new direction in field space whose potential has a steep direction, this direction overlaps with ones from other more conventional sources, and do not affect our no-go result. The direction of steepest ascent for the D-string oscillator modes is derived in Appendix C, and is shown in figure 1. However if massless D-string modes are also allowed in the string theory spectrum, they can lift the runaway direction in conjunction with other modes, as has been argued in [21].

Another possible source that provides an effective potential is the NS5-brane.<sup>4</sup> Its tension behaves as  $T_5^F \propto g_s^{-2}$ , so an NS5-brane wrapping the internal manifold corresponds to  $\alpha = 2$ ,  $\hat{\omega} = \frac{-5}{n}$ ,  $\omega = 0$ ; this results in the coupling coefficients

$$\mu_5^F = -\left(\frac{n}{2} - 2\right) \frac{1}{\sqrt{n+2}} \text{ and } \nu_5^F = (10 - n) \sqrt{\frac{2}{n(n+2)}} \quad (3.23)$$

Thus, for the case  $n = 6$ , the potential rises maximally in the direction  $\psi = -\frac{1}{\sqrt{3}}\phi$ , corresponding to the angle  $\theta_{F5} = 150^\circ$  in figure 1. This does not coincide with the runaway direction identified in figure (1), but is close enough so that NS5-branes in conjunction with strings or D-branes can stabilize all the moduli.

## 4. Adding Superpotentials

Since it is not possible to fully stabilize all moduli using the D-brane/string gas, we investigate the dynamics of a system in which the brane gas is present simultaneously with an external stabilization mechanism. A typical potential which could arise in string-motivated supergravity theories is the one which is generated by gaugino condensation in an  $SU(N)$  gauge sector. Although it might seem redundant to consider partial modulus stabilization by a brane gas when there is already a potential at zero density, there could actually be several benefits: for example, the brane gas can prevent the problem of the moduli overshooting the desired minimum [13], as we investigate in this section.

### 4.1 Gaugino Condensation Potential

We briefly review the derivation of the nonperturbative gaugino condensate potential in low-energy effective supergravity, starting with the 10 dimensional spacetime which is assumed to be a product of 4D noncompact external spacetime and a 6D compact internal

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<sup>3</sup>See Appendix C for details.

<sup>4</sup>We thank Ali Kaya for pointing this out to us.

manifold. We limit our present discussion to the dynamics of the radion,  $\psi(x)$ . A similar discussion should apply for more than one moduli field, but for simplicity we assume that all other moduli (*i.e.*, complex structure and dilaton) have been stabilized. The radion appears in the full metric as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + e^{2\psi} g_{mn} dy^m dy^n \quad (4.1)$$

In supergravity, the radion is the real part of a chiral field,

$$T = X + iY \equiv e^{4\psi} + iY \quad (4.2)$$

Dimensional reduction of the supergravity action yields an effective four dimensional theory of gravity coupled to the complex scalar field  $T(x)$

$$S = M_p^2 \int d^4x \sqrt{-g} \left[ \frac{R}{2} + \mathcal{K}_{T\bar{T}} \partial_\mu T \partial^\mu \bar{T} - e^\mathcal{K} (\mathcal{K}^{T\bar{T}} D_T \mathcal{W} \overline{D_T \mathcal{W}} - 3|\mathcal{W}|^2) \right] \quad (4.3)$$

where  $\mathcal{K}(T, \bar{T})$  and  $\mathcal{W}(T, \bar{T})$  are the Kähler potential and superpotential respectively, while  $\mathcal{K}_{T\bar{T}}$  is the Kähler metric given by

$$\mathcal{K}_{T\bar{T}} = \frac{\partial^2 \mathcal{K}}{\partial T \partial \bar{T}} \quad (4.4)$$

We have also performed a conformal transformation of the four dimensional metric:

$$g_{\mu\nu} \rightarrow e^{n\psi} g_{\mu\nu} = e^{6\psi} g_{\mu\nu} \quad (4.5)$$

The kinetic and potential terms for  $T$  are computed from  $\mathcal{K}(T, \bar{T})$  and  $\mathcal{W}(T, \bar{T})$ , where the Kähler potential for  $T$  is  $\mathcal{K} = -3 \ln[T + \bar{T}]$  while as in [14] we use the superpotential

$$\mathcal{W} = \mathcal{W}_0 + A e^{-aT} \quad (4.6)$$

which would be obtained through gaugino condensation in a theory with a simple gauge group. For instance, for  $SU(N)$ ,  $a = 2\pi/N$ . The constant term  $\mathcal{W}_0$  represents the effective superpotential due to any fields that have been fixed already [23], such as the dilaton and complex structure moduli.<sup>5</sup>

The scalar-tensor action then reads

$$S = M_p^2 \int d^4x \sqrt{-g} \left[ \frac{R}{2} + K - V \right] \quad (4.7)$$

where the kinetic ( $K$ ) and potential ( $V$ ) terms are given by

$$K = -3 \frac{\partial_\mu T \partial^\mu \bar{T}}{|T + \bar{T}|^2} = -12 \partial_\mu \psi \partial^\mu \psi - \frac{3}{4} e^{-8\psi} \partial_\mu Y \partial^\mu Y$$

---

<sup>5</sup>Although recent authors have pointed out that a proper construction of the KKLT mechanism includes other non-perturbative contributions to the Kähler potential, we are primarily concerned with addressing the overshoot problem which still exists despite their inclusion [24, 25]. That is, the mechanism proposed still provides an attractor solution despite changes to the form of the Kähler potential.

and

$$V = \frac{E}{X^\alpha} + \frac{1}{6X^2} [aA^2(aX + 3)e^{-2aX} + 3W_0Aae^{-aX} \cos(aY)] \quad (4.8)$$

To arrive at the potential (4.8) we have also included the potential energy coming from an anti-D3 brane (first term) as in [14], which is needed in order to have a nonnegative vacuum energy density. The coefficient  $E$  is a function of the tension of the brane  $T_3$  and of the warp factor, if there are warped throats [26] on the Calabi-Yau manifold. The exponent  $\alpha$  is either  $\alpha = 2$  if the anti-D3 branes are sitting at the end of a warped throat. Otherwise  $\alpha = 3$  corresponding to the unwarped region. If a warped region exists, it is energetically preferred.

The imaginary part of the Kähler modulus, the axion  $Y$ , has stable minima at  $Y = (2n + 1)\pi/a$  (assuming  $W_0Aa > 0$ ). We will integrate this field out and focus on the dynamics of the radion, whose kinetic term is<sup>6</sup>  $12M_p^2(\partial\psi)^2$ , and whose potential becomes

$$V = Ee^{-4\alpha\psi} - \frac{1}{2} (W_0Aae^{-aX} - aA^2e^{-2aX}) e^{-8\psi} + \frac{1}{6} a^2 A^2 e^{-2aX} e^{-4\psi} \quad (4.9)$$

It is convenient to rescale  $\psi \rightarrow \psi/\sqrt{24}$  so that the kinetic term is canonically normalized. The action becomes

$$S = M_p^2 \int d^4x \sqrt{-g} \left[ \frac{R}{2} - \frac{(\partial\psi)^2}{2} - \left( Ee^{-2\alpha_1\psi} - \frac{1}{2} (W_0Aae^{-aX} - aA^2e^{-2aX}) e^{-2\alpha_2\psi} + \frac{1}{6} a^2 A^2 e^{-2aX} e^{-2\alpha_3\psi} \right) \right] \quad (4.10)$$

where

$$\alpha_1 = \frac{\alpha}{\sqrt{6}}, \quad \alpha_2 = \frac{2}{\sqrt{6}}, \quad \alpha_3 = \frac{1}{\sqrt{6}}, \quad X = e^{2\alpha_3\psi} \quad (4.11)$$

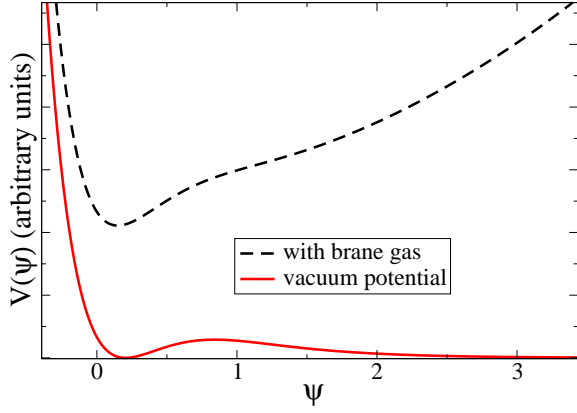
The potential has three distinct regions (see figure 2, solid curve). (1) For  $\psi$  large and negative  $V(\psi)$  is dominated by the antibrane contribution, if  $\alpha = 3$ , or by a combination of the antibrane term and the term proportional to  $e^{-2\alpha_2\psi}$  if  $\alpha = 2$ . In either case, the potential is to a good approximation a pure exponential in  $\psi$ , which will be relevant for the analytic solutions we discuss in the next subsection. (2) For  $\psi \sim 1$ , the different terms in the potential are comparable, creating a minimum at  $\psi_{\min}$ , followed by a potential barrier at  $\psi_{\max}$ . (3) For  $\psi \gg 1$  the antibrane term again dominates, since the other terms are exponentially suppressed by  $e^{-aX}$ .

## 4.2 Attractor solution with brane gases

We now consider the effect of augmenting the vacuum potential in (4.10) with the contribution from a brane gas. Without the brane gas, the dynamics of the radion depend sensitively on the initial conditions. If we start with  $\psi > \psi_{\max}$  (the position of the bump in the potential),  $\psi$  runs to infinity, where the extra dimensions are decompactified.

Generically one might expect the radion to start closer to the Planck size with  $\psi < 0$ , so that there is a possibility of reaching the stable minimum at  $\psi = \psi_{\max}$ .

<sup>6</sup>The kinetic part of the action for the radion can also be derived directly from the Einstein-Hilbert action  $S_{10} = \int d^{10}x \sqrt{-\hat{g}} \hat{R}$  contained in the full 10D supergravity action. Using a consistent dimensional reduction ansatz for the metric of the form (4.1) one obtains  $S_4 = \int d^4x \sqrt{-g} e^{n\psi} [R - n(n-1)(\partial\psi)^2 + \dots]$ . A conformal transformation precisely of the form (4.5) is needed to convert the action  $S_4$  to Einstein frame, from which one recovers the kinetic piece of the radion given here.



**Figure 2:** Radion potential with vacuum gaugino condensate potential (solid line) and potential at nonzero brane gas density.

Since the vacuum potential in the region  $\psi < 0$  is well approximated by an exponential, the radion quickly reaches the attractor solution discussed in [27]; it tracks the minimum formed between the exponential potential and the rising part of the “brane-gas potential,” shown as the dashed line in figure 2. This attractor behavior washes out the effect of initial conditions. As long as the attractor is reached before the field has passed the position of the minimum, this will allow  $\psi$  to settle into the minimum and avoid the overshoot problem.

Let us recapitulate the details of the attractor solution. The rising part of the brane-gas potential originates from the winding modes of  $p$ -branes with  $p > 3$ . In this region the Friedmann equation and the equation of motion for  $\psi$  read (in  $M_p = 1$  units)

$$H^2 \cong \frac{1}{3} \left( \frac{1}{2} \dot{\psi}^2 + E e^{-2\alpha_1 \psi} + \rho_p e^{2\nu_p \psi} \right) \quad (4.12)$$

with  $\rho_p = \rho_p^0 \left( \frac{a}{a_0} \right)^{-3}$  and

$$\ddot{\psi} + 3H\dot{\psi} \cong 2 \left( \alpha_1 E e^{-2\alpha_1 \psi} + \nu_p \rho_p e^{2\nu_p \psi} \right) \quad (4.13)$$

respectively. The exponents  $\nu_p$  for  $p$ -branes’ coupling to the canonically normalized radion were derived in the previous section,

$$2\nu_p = \sqrt{\frac{2}{n(n+2)}} \left( p - \frac{n}{2} \right) = \sqrt{\frac{1}{24}} (p - 3) \quad (4.14)$$

where  $n = 6$  is the number of extra dimensions. This kind of system was studied in [28], where it was shown that there exist tracking solutions in which the energy of the scalar field tracks that of the branes:

$$e^\psi = \left[ \frac{E}{\rho_p} \left( \frac{\alpha_1(\alpha_1 + \nu_p) - 3/4}{\nu_p(\nu_p + \alpha_1) + 3/8} \right) \right]^{\frac{1}{2(\alpha_1 + \nu_p)}} \equiv \left[ \frac{E}{\rho_p} r \right]^{\frac{1}{2(\alpha_1 + \nu_p)}} \quad (4.15)$$

This relation implies that both the potential and kinetic energy of the radion remain proportional to the energy density of the branes,

$$V(\psi) = r^{-1} \rho_p e^{2\nu_p \psi} = \frac{8(\nu_p + \alpha_1)^2 - 3}{3(1+r)} K \quad (4.16)$$

The steepest brane-induced effective potential occurs for the maximal value of  $p$ ,  $p = 6$ ; this provides the greatest resistance to expansion of the internal manifold and will be the most effective case for avoiding the overshoot problem. Eq. (4.16) also shows that for  $p = 6$  the ratio of kinetic to potential energy is minimized. For example, if  $\alpha = 2$  and  $p = 6$  we have  $\alpha_1 = \sqrt{2/3}$ ,  $2\nu_6 = \sqrt{3/8}$ , leading to  $K/V = 12/23$ .

In passing we note that such tracking solutions correspond to a power law expansion of the universe

$$a(t) = a_0 \left( \frac{t}{t_0} \right)^{(2/3)(1+\nu_p/\alpha_1)} = a_0 \left( \frac{t}{t_0} \right)^{11/12} \quad (4.17)$$

The universe does not accelerate during this phase. However, as was found in [28], when the analysis is carried out including the dilaton, acceleration can be obtained.

### 4.3 Addressing the Overshoot Problem

The above discussion implies that the overshoot problem will be avoided in the presence of a brane gas so long as the attractor solution can be reached. This means that for a given initial value of  $\psi$ , the initial energy density in the brane gas,  $\rho_p e^{2\nu_p \psi}$ , must be sufficiently large. If not, the brane density is diluted too quickly by the expansion of the universe and the system evolves according to the vacuum potential.

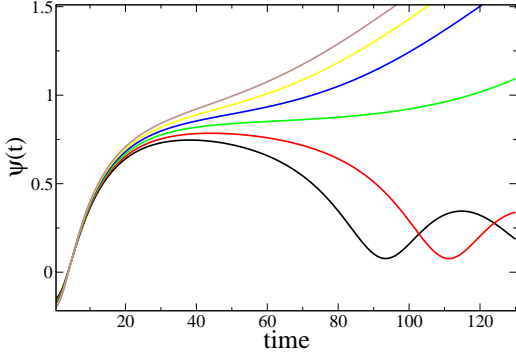
We have confirmed these expectations by numerically integrating the coupled system of Friedmann and radion equations, which we illustrate with a specific example. In the potential (4.8) we consider an antibrane in a warped throat, with  $\alpha = 2$ . Its tension is tuned to give a Minkowski minimum as shown in figure 2, which illustrates the case where  $E = 0.00889$ ,  $a = 2.1$ ,  $A = 0.9$ , and  $W_0 = 0.25$ . We first verified that indeed this potential suffers from an overshoot problem, shown in figure 3. Starting from an initial condition  $\psi \lesssim -0.17$ , the field runs away to  $\infty$ .

Interestingly, overshooting can be prevented by initial brane densities which are many orders of magnitude smaller than the initial potential energy of the radion. Figure 4 shows the evolution starting from exponentially large initial radion potential energy, with  $\psi_0 = -100$  and  $p = 6$ , for several initial brane densities, parametrized by  $\zeta = \rho_p e^{2\nu_p \psi} / V_0(\psi_0)$ , where  $V_0$  is the potential of the radion alone, excluding the brane gas contribution. The result shows that even for initial brane gas energy densities which are only  $10^{-18} V_0(\psi_0)$ , overshoot can be prevented. For different initial values, the exponent  $\log_{10}(\zeta)$  scales linearly with  $\psi_0$ . This behavior can be understood analytically, as shown in Appendix D. The minimum required value of  $\zeta$  is given by

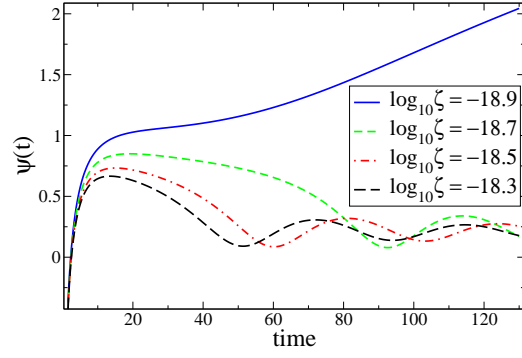
$$\log_{10} \zeta \approx -0.43 \left[ 2\alpha_1 \psi_0 \left( 1 - \frac{3 - 4\nu_p \alpha_1}{4\alpha_1^2} \right) \right] \quad (4.18)$$

The intuitive explanation for this result is that the radion energy initially falls more quickly than that of the brane gas. What counts is not the initial ratio of brane gas to potential energy; rather it is the ratio at the time when  $\psi$  is close to its nontrivial minimum. This mechanism has been pointed out in [29] (see also [30, 31]) as a generic way of solving the overshoot problem, using general sources of energy density. Brane gas cosmology provides a concrete setting where this idea can be used advantageously.

When the modulus has reached its stable minimum, we are still left with a gas of branes, whose energy density is comparable to the energy density in the scalar fields; otherwise the brane gas would not be effective in slowing the rolling of the modulus. At the bottom of its potential, the scalar field oscillates and its energy density redshifts as  $a^{-3}$  just like the brane gas. The result is a matter dominated universe. We must assume that



**Figure 3:** Evolution of  $\psi$  without brane gas, for several initial values  $\psi_0 = -0.15, -0.16, \dots, -0.2$ , illustrating overshoot.



**Figure 4:** Solutions with  $\psi_0 = -100$  and different initial densities of brane gas, near the borderline of overshooting.

inflation begins some time after this in order to dilute the branes and reheat the universe. Work on smoothly connecting the modulus stabilization with the beginning of inflation is in progress.

## 5. Conclusions

In this paper we used dimensional reduction to derive the effective action for a gas of strings and  $p$ -branes, giving a contribution to the effective potential for the radion and dilaton. In a gas of strings only, this potential could stabilize the radion provided there was only one extra dimension, but not the dilaton. Including  $p$ -branes allows for the stabilization of either the dilaton or radion if  $p > \frac{d}{2}$ . However, the brane gas is insufficient for stabilizing both moduli simultaneously, for the type II strings we consider, which have no massless winding modes. Rather, only a linear combination of the moduli can be stabilized by the brane gas.

It thus seems likely that external potentials are needed for modulus stabilization. However the brane gas can still play an interesting role in helping the moduli settle into their typically shallow minima, avoiding the overshoot problem. An attractive feature of this mechanism is that the brane gas can initially be many orders of magnitude smaller in energy density than the potential energy of the moduli and still be effective in slowing the rolling of the moduli, since the brane gas energy redshifts more slowly. There is therefore no need for finely-tuned initial conditions.

In this work we have ignored quantum corrections, as well as higher-derivative corrections to the dilaton gravity action. The first approximation is justified for weak string coupling,  $g_s = e^\phi \ll 1$ . In this regard, the runaway direction found in Section 2 corresponds to  $\phi \rightarrow -\infty$ , showing that quantum corrections cannot lift this flat direction at large field values. Of course it is possible that such corrections could lead to a metastable minimum along the flat direction, which would be a loophole in our no-go result.

## Acknowledgments

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## Note Added

As this work was being finished, similar results were given in [32]. Our results were presented the week before, at the McGill Workshop on String Gas Cosmology, 30 April 2005.

## A. Dimensional Reduction

We provide here a brief review of the standard dimensional-reduction procedure, following the procedure of [11, 33]. Our starting point is  $D$ -dimensional dilaton-gravity together with a generic contribution of gas. This system is described by

$$S_{II} = \frac{1}{2\kappa^2} \int d^D x \sqrt{-G} e^{-2\phi} \left( R + 4G^{MN} \nabla_M \phi \nabla_N \phi - \frac{1}{12} H_{\mu\nu\alpha} H^{\mu\nu\alpha} \right) \quad (\text{A.1})$$

$$S_m = \int d^D x \sqrt{-G} e^{-\alpha\phi} \rho, \quad \rho = \sum_i \rho_i a^{-d(1+\omega_i)} b^{-n(1+\hat{\omega}_i)}, \quad (\text{A.2})$$

for some initial density  $\rho_i$ . The dimensional reduction procedure will focus on the string action (A.1); but, by tracking the transformation rules, we can later also reduce the matter components. We obtain an effective theory of BGC by first transforming the string action (A.1) to the Einstein frame through the conformal transformations [34]

$$\begin{aligned} G_{MN} &\rightarrow \tilde{G}_{MN} = \Omega^2 G_{MN}, \quad \Omega = e^{-A\phi}, \quad A = \frac{2}{D-2} \\ R &\rightarrow \tilde{R} : R = e^{-2A\phi} \tilde{R} - 2(D-1)e^{-A\phi} \left( e^{-A\phi} \right)_{;MN} \tilde{G}^{MN} \\ &\quad - (D-1)(D-4) \left( e^{-A\phi} \right)_{;M} \left( e^{-A\phi} \right)_{;N} \tilde{G}^{MN} \\ \phi &\rightarrow \tilde{\phi} = \sqrt{2A} \phi \end{aligned} \quad (\text{A.3})$$

to obtain

$$S \rightarrow \tilde{S} = \frac{1}{2\kappa^2} \int d^D x \sqrt{\tilde{G}} \left\{ \tilde{R} - \tilde{G}^{MN} \nabla_M \tilde{\phi} \nabla_N \tilde{\phi} \right\}, \quad (\text{A.4})$$

where  $\tilde{\phi}$  is the canonically-normalized dilaton, and we have ignored flux contributions. We dimensionally reduce the action by integrating out the extra dimensions [11, 33]. To perform this last step we consider a string-frame metric of the form (2.3), split into  $d$  large directions described by  $g_{\mu\nu}$  and  $n$  compact directions described by  $\gamma_{mn}$ . For simplicity, we consider the geometry of the extra dimensions to be that of a torus, thus  $R[\gamma_{mn}] = 0$ . We use the following relations to isolate the scale-factor dependence on the extra-dimensions [11, 33, 34]

$$\sqrt{-\tilde{G}} = \tilde{b}^n \sqrt{-\tilde{g}} \quad (\text{A.5})$$

$$\tilde{R} = \tilde{R}[\tilde{G}_{MN}] = \tilde{R}[\tilde{g}_{\mu\nu}] - 2n\tilde{b}^{-1} \tilde{g}^{\mu\nu} \tilde{\nabla}_\mu \tilde{\nabla}_\nu \tilde{b} - n(n-1) \tilde{b}^{-2} \tilde{g}^{\mu\nu} \tilde{\nabla}_\mu \tilde{b} \tilde{\nabla}_\nu \tilde{b}, \quad (\text{A.6})$$

where, again,  $R[\gamma_{mn}] = 0$ ,  $n$  and  $\tilde{b}(x^\mu)$  are the number and scale factor corresponding to the extra dimensions, and  $\tilde{g}_{\mu\nu}$  is the metric of the non-compact directions. Since none of the terms in the action depend explicitly on the coordinates from the  $n$  extra dimensions, we integrate over these directions to get the low energy effective action of the  $d+1$ -dimensional theory

$$S_{eff} = \frac{V_n}{2\kappa^2} \int d^{d+1}x \sqrt{-\tilde{g}} \left[ \left( \tilde{b}^n d\tilde{R}[\tilde{g}_{\mu\nu}] - 2n\tilde{b}^{n-1} \tilde{g}^{\mu\nu} \tilde{\nabla}_\mu \tilde{\nabla}_\nu \tilde{b} - n(n-1) \tilde{b}^{n-2} \tilde{g}^{\mu\nu} \tilde{\nabla}_\mu \tilde{b} \tilde{\nabla}_\nu \tilde{b} \right) - \tilde{b}^n \tilde{g}^{\mu\nu} \tilde{\nabla}_\mu \tilde{\phi} \tilde{\nabla}_\nu \tilde{\phi} \right], \quad (\text{A.7})$$

where  $V_n \equiv \int d^n y \sqrt{\gamma}$  is the spatial volume of the  $n$  extra dimensions under unit scaling ( $\tilde{b} = 1$ ).

A second conformal transformation and field redefinition of the action (A.7) is necessary to obtain the canonical form of the Einstein-Hilbert action. The conformal transformation reuses the identities (A.3) with

$$\bar{g}_{\mu\nu} = \tilde{b}^n \tilde{g}_{\mu\nu} \equiv e^{\sqrt{B}\tilde{\psi}} \tilde{g}_{\mu\nu}, \quad (\text{A.8})$$

resulting in

$$S_{eff} = \frac{V_n}{2\kappa^2} \int d^{d+1}x \sqrt{-\bar{g}} \left( R[\bar{g}_{\mu\nu}] - \bar{g}^{\mu\nu} \bar{\nabla}_\mu \tilde{\psi} \bar{\nabla}_\nu \tilde{\psi} - \bar{g}^{\mu\nu} \bar{\nabla}_\mu \tilde{\phi} \bar{\nabla}_\nu \tilde{\phi} \right), \quad (\text{A.9})$$

where  $B = \frac{d-1}{n(d+n-1)}$ . Finally, the system is canonically normalized by identifying the 4D Planck mass as  $M_p^2 \equiv \frac{V_n}{\kappa^2}$ , and by rescaling the fields as

$$\psi = M_p \tilde{\psi}, \quad \varphi = M_p \tilde{\phi} \quad (\text{A.10})$$

$$\Rightarrow S_{eff} = \int d^{d+1}x \sqrt{-\bar{g}} \left( \frac{M_p^2}{2} R[\bar{g}_{\mu\nu}] - \frac{1}{2} \bar{g}^{\mu\nu} \bar{\nabla}_\mu \psi \bar{\nabla}_\nu \psi - \frac{1}{2} \bar{g}^{\mu\nu} \bar{\nabla}_\mu \varphi \bar{\nabla}_\nu \varphi \right). \quad (\text{A.11})$$

The net effect of these transformations is to rescale the scale factors and dilaton as

$$\begin{aligned} \sqrt{-G} &\rightarrow \sqrt{-\bar{g}} e^{D\sqrt{\frac{A}{2}}\frac{\varphi}{M_p}} e^{n\sqrt{B}\frac{\psi}{M_p}} \\ a(t) &\rightarrow \bar{a}(t) = e^{\frac{n}{d-1}\sqrt{B}\frac{\psi}{M_p}} e^{-\sqrt{A/2}\frac{\varphi}{M_p}} a(t) \\ b(t) &\rightarrow \tilde{b}(t) = e^{\sqrt{B}\frac{\psi}{M_p}} e^{-\sqrt{A/2}\frac{\varphi}{M_p}} b(t) \\ \phi(t) &\rightarrow \varphi(t) = \sqrt{2A}M_p \phi(t), \end{aligned} \quad (\text{A.12})$$

Employing the above expressions, we may now express the contribution of a source behaving as

$$\rho = \rho_i a^{-d(1+\omega_i)} b^{-n(1+\hat{\omega}_i)} \quad (\text{A.13})$$

in the  $D$  dimensional string frame, through the effective matter-action

$$\begin{aligned} S_{effm} &= \int d^{d+1}x \sqrt{-\bar{g}} e^{-\alpha\sqrt{\frac{1}{2A}}\varphi} \bar{\rho} \\ &= \int d^{d+1}x \sqrt{-\bar{g}} \rho_i e^{(-\hat{\omega}_i + \frac{d}{d-1}(\omega_i - \frac{1}{d}))\sqrt{\frac{(d-1)n}{(d+n-1)}}\psi} e^{(-d\omega_i - n\hat{\omega}_i + 1 - \alpha_i \frac{d+n-1}{2})\sqrt{\frac{1}{d+n-1}}\varphi} \bar{a}^{-d(1+\omega_i)} \\ &\equiv \int d^{d+1}x \sqrt{-\bar{g}} \rho_i e^{2(\mu_i \varphi + \nu_i \psi)} \end{aligned} \quad (\text{A.14})$$

with  $M_p = 1$ . The original theory of dilaton gravity together with string and brane sources can now be interpreted as a theory of Einstein gravity together with sources, plus two scalar fields corresponding to the dilaton ( $\varphi$ ) and the moduli field ( $\psi$ ), this is the action of equation (A.11). As well, the source term (equation A.2) now acts like an effective potential for the two scalar fields. The inclusion of different excited states will provide different effective potentials, and this freedom can be exploited in the search for a moduli-stabilizing potential.

## B. Equations of state

In this section we derive the equations of state and the resultant coefficients for the brane-gas effective potential. Using the metric-ansatz (2.3), we derive the gas pressure from the thermodynamic relation.

$$P_a = - \left. \frac{\delta E}{\delta V} \right|_{b=const.} \quad (\text{B.1})$$

The volume is given by  $V = \sqrt{-G_s} = a^d b^n$ , while energy contributions are generically of the form  $E = a^j b^k = (a^d)^{\frac{j}{d}} (b^n)^{\frac{k}{n}}$ , so that

$$\delta V = b^n \delta(a^d) + a^d \delta(b^n) \quad (\text{B.2})$$

$$\delta E = \frac{j}{d} \frac{a^j b^k}{a^d} \delta(a^d) + \frac{k}{n} \frac{a^j b^k}{b^n} \delta(b^n) \quad (\text{B.3})$$

$$\Rightarrow P_a = - \left. \frac{\delta E}{\delta V} \right|_{b=const} = - \frac{j}{d} \frac{a^j b^k}{a^d b^n} \frac{\delta(a^d)}{\delta(a^d)} = \omega \frac{E}{V} = \omega \rho \quad (\text{B.4})$$

$$\Rightarrow P_b = - \left. \frac{\delta E}{\delta V} \right|_{a=const} = - \frac{k}{n} \frac{a^j b^k}{a^d b^n} \frac{\delta(b^n)}{\delta(b^n)} = \hat{\omega} \frac{E}{V} = \hat{\omega} \rho, \quad (\text{B.5})$$

where we have made the identifications  $\omega = -\frac{j}{d}$  and  $\hat{\omega} = -\frac{k}{n}$ . Thus  $E = a^j b^k = a^{-d\omega} b^{-n\hat{\omega}}$ . The existence of winding and momentum modes for strings is a well-known result, and is the reason for the T-duality invariant spectrum of closed strings. Finding an embedding with quantized momentum modes of branes is less subtle because the T-dual of a wrapped brane results in a wrapped brane, not a momentum mode. However, we use the embedding described by Kaya [16], which results in momentum modes in the compact direction with energy given by

$$E_n = \frac{\lambda_n}{b(t)}, \quad (\text{B.6})$$

where  $\lambda_n$  is an unknown eigenvalue for the  $n$ 'th momentum mode (we choose  $\lambda_n > 0$ ). The corresponding pressure due to this brane momentum-mode is

$$P_n = \frac{\lambda_n}{b(t)}, \quad (\text{B.7})$$

which is a positive quantity.

### C. D-string oscillator modes

For completeness we consider the naive quantization of D-strings, in case these modes could affect the no-go result for simultaneous stabilization of the dilaton and radion. Ignoring the 2-form gauge field that couples to the D-string, the spectrum of the D-strings looks identical to that of F-strings except for the replacement

$$M_s \longrightarrow M'_s = e^{-\varphi/2} M_s \quad (\text{C.1})$$

The rescaling is again due to the dilaton coupling present in the DBI action for the D-strings. Provided we ignore  $N_{\text{osc}} < 0$  modes, again it is sufficient to consider only the “pure” modes. A straight forward computation yields the following source actions

$$S_{\text{D,mom}} = \int d^D x \sqrt{-\hat{g}} a^{-3} b^{-(n+1)} \quad (\text{C.2})$$

$$S_{\text{D,osc}} = \int d^D x \sqrt{-\hat{g}} e^{-\varphi/2} a^{-3} b^{-n} \quad (\text{C.3})$$

and

$$S_{\text{D,wind}} = \int d^D x \sqrt{-\hat{g}} e^{-\varphi} a^{-3} b^{-(n-1)} . \quad (\text{C.4})$$

The D-string momentum modes looks identical to those of the F-string momentum modes, and yield no new effect. The winding modes are the same as those obtained for D1-branes, which have already been considered. The only qualitatively new contribution comes from the oscillatory D-string modes. Substituting  $\alpha = 1/2$  and  $\hat{\omega} = 0$  in (3.5) one finds

$$\theta_{\text{D,osc}} = -\frac{\pi}{2} - \frac{\pi}{6} \quad (\text{C.5})$$

Again, this fails to stabilize the runaway direction.

### D. Solving the overshoot problem

One can analytically estimate of what must be the initial ratio of energy densities in the brane gas and radion in order to solve the overshoot problem. If the initial energy density of branes is much smaller than the potential energy of the radion, the dynamical equations will be given by

$$H^2 \cong \frac{1}{3} \left( \frac{1}{2} \dot{\psi}^2 + E e^{-2\alpha_1 \psi} \right) \quad (\text{D.1})$$

and

$$\ddot{\psi} + 3H\dot{\psi} \cong 2\alpha_1 E e^{-2\alpha_1 \psi} \quad (\text{D.2})$$

The radion rolls freely down the exponential potential and exact solutions are known [35]:

$$a \sim t^{1/2\alpha_1^2} \text{ and } e^\psi \sim t^{1/\alpha_1} \sim a^{2\alpha_1} \quad (\text{D.3})$$

Thus the energy densities of the brane and the radion redshift in this non-tracking phase as

$$\rho_p e^{2\nu_p \psi} \sim a^{-(3-4\nu_p \alpha_1)} \text{ while } V(\psi) \sim a^{-4\alpha_1^2} \quad (\text{D.4})$$

Thus as long as

$$3 - 4\nu_p\alpha_1 < 4\alpha_1^2 \quad (\text{D.5})$$

the brane energy density will catch up with the potential energy of the radion. We can calculate when this happens. The ratio of brane energy to radion potential energy is

$$\frac{\rho_p e^{2\nu_p\psi}}{V(\psi)} \sim a^{4\nu_p\alpha_1 + 4\alpha_1^2 - 3} \quad (\text{D.6})$$

and we want that this ratio to be  $\mathcal{O}(1)$ , by the time the radion rolls to the minimum. Hence we need to start with an initial ratio such that

$$\zeta \equiv \frac{\rho_{p0} e^{2\nu_p\psi_0}}{V(\psi_0)} = \left( \frac{a_0}{a_{min}} \right)^{4\nu_p\alpha_1 + 4\alpha_1^2 - 3} \quad (\text{D.7})$$

where

$$\frac{V(\psi_0)}{V(\psi_{min})} = e^{-2\alpha_1\psi_0} = \left( \frac{a_0}{a_{min}} \right)^{-4\alpha_1^2} \quad (\text{D.8})$$

From (D.7) and (D.8) we find

$$\zeta = \exp \left[ 2\alpha_1\psi_0 \left( 1 - \frac{3 - 4\nu_p\alpha_1}{4\alpha_1^2} \right) \right] \Rightarrow \log_{10} \zeta \approx -0.43 \left[ 2\alpha_1\psi_0 \left( 1 - \frac{3 - 4\nu_p\alpha_1}{4\alpha_1^2} \right) \right] \quad (\text{D.9})$$

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